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LETTER TO THE EDITOR

Spatial properties of quasi-stationary gaussian optical fields

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Abstract. The spatial coherence properties of quasi-stationary gaussian optical fields are discussed using a terminology that shows the importance of coherence functions of order $N \neq M$. It is shown that this field is spatially less coherent than a thermal field.

Generalizations of gaussian optical fields have been considered by Picinbono and Rousseau (1970). The class of 'quasi-stationary gaussian' optical fields has been introduced and it has been shown that it is more chaotic (or incoherent) than the stationary gaussian (thermal) field (Picinbono 1969). It has also been pointed out that this new class is physically obtainable and in fact it appears in some experimental situations. In this communication, we investigate some of the spatial properties of the quasi-stationary gaussian fields. We show that this class is spatially (as well as temporally) more chaotic than the thermal field. We also reformulate the Picinbono–Rousseau results in a form that shows the importance of coherence functions of odd orders $N \neq M$ (Mandel and Wolf 1965).

We start be reviewing some definitions pertinent to the present development.

Stationary field. An optical field represented by its analytic signal $V(\boldsymbol{x})$ at the space–time point $\boldsymbol{x} = (\boldsymbol{r},t)$ is said to be stationary of order (N,M) if the coherence function (Mandel and Wolf 1965)

$$G^{N,M}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \ldots, \boldsymbol{x}_{N+M}) = \langle V^*(\boldsymbol{x}_1) V^*(\boldsymbol{x}_2) \ldots V^*(\boldsymbol{x}_N) V(\boldsymbol{x}_{N+1}) \ldots V(\boldsymbol{x}_{N+M})\rangle,$$

is independent of shifts in the time origin. If the field is band-limited with bandwidth $\Delta\omega$ and a central frequency ω_0 then (Mandel and Mehta 1969)

$$G^{N,M} = 0, \qquad \frac{|N-M|}{N+M} \geqslant \frac{1}{2}\frac{\Delta\omega}{\omega_0}. \tag{1}$$

This condition is always satisfied for $N \neq M$, unless M and N are very large numbers. In particular, $G^{0,2} = G^{2,0} = 0$.

Complex circular gaussian field. An optical field represented by its analytic signal $V(\boldsymbol{x})$ is said to be gaussian if $V(\boldsymbol{x})$ is a complex gaussian stochastic process, ie, whose real and imaginary parts are jointly gaussian. Such processes are characterized by an expansion which in the order (2,2) takes the form

$$\begin{aligned} G^{2,2}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{x}_3, \boldsymbol{x}_4) &= G^{1,1}(\boldsymbol{x}_1; \boldsymbol{x}_3)G^{1,1}(\boldsymbol{x}_2; \boldsymbol{x}_4) + G^{1,1}(\boldsymbol{x}_1; \boldsymbol{x}_4)G^{1,1}(\boldsymbol{x}_2; \boldsymbol{x}_3) \\ &\quad + G^{2,0}(\boldsymbol{x}_1, \boldsymbol{x}_2)G^{0,2}(\boldsymbol{x}_3, \boldsymbol{x}_4), \end{aligned} \tag{2}$$

where $V(\boldsymbol{x})$ is assumed to have zero mean. This optical field is said to be complex circular gaussian (or thermal) if, moreover, it is stationary. In this case, (1) applies and the expansion (2) becomes the well known formula (Reed 1962)

$$G^{2,2}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{x}_3, \boldsymbol{x}_4) = G^{1,1}(\boldsymbol{x}_1; \boldsymbol{x}_3)G^{1,1}(\boldsymbol{x}_2; \boldsymbol{x}_4) + G^{1,1}(\boldsymbol{x}_1; \boldsymbol{x}_4)G^{1,1}(\boldsymbol{x}_2; \boldsymbol{x}_3). \tag{3}$$

We emphasize that stationarity of the field is a necessary condition for the validity of the often used expansion (3).

Quasi-stationary gaussian field. An optical field whose analytic signal is written in the form $V(\boldsymbol{x}) = A(\boldsymbol{x}) \exp(-j\omega_0 t)$ is said to be quasi-stationary gaussian (QSG) if its amplitude $A(\boldsymbol{x})$ is a stationary band-limited complex gaussian process. This field has the following properties:

(i) It is stationary for orders $N = M$, but nonstationary for orders $N \neq M$.

(ii) Its coherence functions of order $N \neq M$ do not vanish as in the stationary case. Actually the values of these functions can be taken as a measure of the degree of nonstationarity.

(iii) The expansion (2) is valid but that of (3) is not.

(iv) The necessary and sufficient condition for this field to become fully stationary is that $G^{N,M} = 0 \ \forall\ N \neq M$. In such a case the expansion (2) becomes that of a complex circular gaussian field (3).

Intensity correlation properties of QSG fields.

The second order intensity coherence function at two points in space $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and time delay $t_2 - t_1 = \tau$,

$$G_I(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau) = G^{2,2}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{x}_1, \boldsymbol{x}_2)$$

has great importance in the fields of astronomy (Hanbury Brown 1968), and intensity fluctuation spectroscopy (Pike and Jakeman 1973). For a quasi-stationary optical field we can use the expansion (2) and write

$$G_I(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau) = I(\boldsymbol{r}_1)I(\boldsymbol{r}_2) + |G^{1,1}(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau)|^2 + |G^{0,2}(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau)|^2. \tag{4}$$

where $I(\boldsymbol{r}) = G^{1,1}(\boldsymbol{x}, \boldsymbol{x})$ is the field's intensity. We note that when the field is stationary, $G^{0,2} = 0$ and we recover the celebrated expansion of a complex circular gaussian process

$$G_I(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau) = I(\boldsymbol{r}_1)I(\boldsymbol{r}_2) + |G^{1,1}(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau)|^2. \tag{5}$$

Also if the field has a real amplitude, then $|G^{0,2}| = |G^{1,1}|$ and equation (4) becomes

$$G_I(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau) = I(\boldsymbol{r}_1)I(\boldsymbol{r}_2) + 2|G^{1,1}(\boldsymbol{r}_1, \boldsymbol{r}_2; \tau)|^2. \tag{6}$$

Picinbono and Rousseau (1970) have obtained an expansion equivalent to (4) (equation (5.2) of their paper). However, this expansion is given in terms of elements of the correlation matrix describing the correlations of the real and imaginary parts of the field amplitude. The present description in terms of coherence functions of odd orders seems to be more straightforward (for example, the condition $G^{0,2} = 0$ is equivalent to equations (2.8) and (2.9) of their paper). Also, the generalization of (2), would enable us to directly find an expansion for amplitude coherence functions of any order (most important of which is the order 2,2 which is often necessary in intensity correlation experiments). Now in order to compare between the degree of spatial intensity correlation of both the QSG field and the thermal field, we use a criterion analogous to that of Picinbono (1969),

$$h = \frac{G_I(\boldsymbol{r}_1 = \boldsymbol{r}_2 = \boldsymbol{r}; 0)}{G_I(\boldsymbol{r}_1 - \boldsymbol{r}_2 = \infty; 0)}. \tag{7}$$

By using (4) and noting that for $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ sufficiently separated, $|G^{1,1}| = |G^{2,0}| = 0$, it follows that

$$h = 2+\epsilon,$$

where

$$0 \leqslant \epsilon = \frac{|G^{2,0}(\boldsymbol{r}, \boldsymbol{r}; 0)|^2}{|G^{1,1}(\boldsymbol{r}, \boldsymbol{r}; 0)|^2} \leqslant 1,$$

hence

$$2 \leqslant h \leqslant 3.$$

Note that for a fully stationary field (thermal) $\epsilon = 0$ and $h = 2$ and for a quasi-stationary field with real amplitude $\epsilon = 1$ and $h = 3$, indicating that the QSG field is spatially more chaotic than the thermal field. This is a result analogous to the temporal property previously reported by Picinbono and Rousseau (1970).

Concluding, we give two examples of QSG fields.

(i) *Mixture of coherent and thermal light.* Consider a thermal light field $V_i(\boldsymbol{x}) = A_i(\boldsymbol{x}) \exp(-\mathrm{j}\omega_0 t)$, mixed with a coherent plane wave of the same central frequency $V_c(\boldsymbol{x}) = A_c \exp(-\mathrm{j}\omega_0 t)$. The mixed light $V(\boldsymbol{x}) = (A_i(\boldsymbol{x}) + A_c) \exp(-\mathrm{j}\omega_0 t)$ is quasi-stationary guassian (because its amplitude is gaussian, stationary and band-limited). In this case, $G^{0,2}(\boldsymbol{x}_1, \boldsymbol{x}_2) = A_c{}^2 \exp[-\mathrm{j}\omega_0(t_1+t_2)] \neq 0$. Yet it should not be concluded that this field is less coherent than a thermal field. This field has an amplitude with nonzero mean and hence the expansion (2) has to be modified. If this is done we reproduce the expansion already known in studies of heterodyne detection of gaussian light (Pike and Jakeman 1973) and get, $1 \leqslant h = 2 - I_c/(I_i + I_c) \leqslant 2$, where I_i and I_c are the intensities of the incoherent and coherent parts of the field.

(ii) *Light scattered from fluids.* The light scattered from susceptibility fluctuations of a fluid target illuminated by an ideal laser is approximately represented by (Cummins and Swinney 1969)

$$V(\boldsymbol{k}, t) \sim \int_v \Delta\chi(\boldsymbol{r}, t) \exp[\mathrm{j}(\boldsymbol{k} - \boldsymbol{k}_0) . \boldsymbol{r}] \, \mathrm{d}^3\boldsymbol{r} \exp(-\mathrm{j}\omega_0 t),$$

where $\Delta\chi(\boldsymbol{r},t)$ represents the susceptibility fluctuation, assumed stationary, homogeneous and gaussian, and $\boldsymbol{k}_0$ and $\boldsymbol{k}$ are wavevectors in the directions of the incident plane wave and the observation point respectively. This field is obviously quasi-stationary gaussian. In order to check the conditions under which the field is fully stationary (and therefore thermal) we calculate

$$G^{2,0}(\boldsymbol{k}_1, \boldsymbol{k}_2; \tau = 0) = \int_v \int_v G_{\Delta\chi}(\boldsymbol{r} - \boldsymbol{r}') \exp[\mathrm{j}(\boldsymbol{k}_1 - \boldsymbol{k}_0) . \boldsymbol{r} + \mathrm{j}(\boldsymbol{k}_2 - \boldsymbol{k}_0) . \boldsymbol{r}'] \, \mathrm{d}^3\boldsymbol{r} \, \mathrm{d}^3\boldsymbol{r}'$$

where $G_{\Delta\chi}(\boldsymbol{r} - \boldsymbol{r}') = \langle \Delta\chi(\boldsymbol{r},t) \Delta\chi(\boldsymbol{r}',t) \rangle$. Assuming that the target size $v^{1/3}$ is much longer than the width of $G_{\Delta\chi}(\boldsymbol{r})$ and changing the variables of integration we can write

$$G^{2,0}(\boldsymbol{k}_1, \boldsymbol{k}_2; 0) \simeq \mathscr{I}_v\left(\frac{\boldsymbol{k}_1 + \boldsymbol{k}_2}{2} - \boldsymbol{k}_0\right) \int_{-\infty}^{\infty} G_{\Delta\chi}(\boldsymbol{r}) \exp\left[\mathrm{j}\left(\frac{\boldsymbol{k}_1 - \boldsymbol{k}_2}{2}\right) . \boldsymbol{r}\right] \mathrm{d}^3\boldsymbol{r}$$

where

$$\mathscr{I}_v(\boldsymbol{k}) = \int_v \exp(\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3\boldsymbol{r}$$

is a function determined by the target dimensions which decreases rapidly as $\boldsymbol{k}$ increases. This determines a solid angle for the observation vector $\frac{1}{2}(\boldsymbol{k}_1+\boldsymbol{k}_2)$ centred around the forward scattering direction $\boldsymbol{k}_0$, outside which $\mathscr{I}_v = 0$, $G^{2,0} = 0$ and hence the field is fully stationary. If the fluctuations correlation length is comparable to the target dimensions (a situation of recent interest) then the problem is not as simple, and quasi-stationarity may extend to wider scattering angles.

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